THE OPEN UNIVERSITY MST204 CG

Mathematics/Science/Technology

MST204 Mathematical Models and Methods

MST204 Course Guide

Introduction

Welcome to the MST204 course! We hope you will enjoy it.

The course is about the use of mathematics to solve real-world problems. Half of it is about how to represent relevant aspects of the real world by means of mathematical *models*; the other half is about mathematical *methods* that are useful in working with these models.

The 'models' half of the course consists of 9 units on Newtonian mechanics (mainly dynamics), 2 units on standard non-mechanical models which can be used for purposes such as natural resource planning and energy saving in the home, and 1 unit devoted to the skills of mathematical modelling. In addition you will be asked to do an extended piece of mathematical modelling yourself.

The 'methods' half of the course consists of 4 units on differential equations and 12 shared among vectors, advanced calculus, matrices and numerical analysis.

A chart at the end of this Guide gives some more information about the interrelationships of the various units.

Like most Open University courses, this one combines the study of correspondence texts with some other activities. Before starting the course you will need to know a little about each. The rest of this Guide contains brief notes about the various activities and components of the course, together with a few additional pieces of information or advice. Cross-references between these notes are denoted by the use of small capital letters (e.g. HANDBOOK, SUMMER SCHOOL). The notes are arranged alphabetically for ease of future reference, but you should read through them *all* now.

Assessment

This comprises 4 components, as follows.

Type	Number	Number substitutable (see below)	Maximum mark	Parts of course covered
TMA	6	2	25	All except MATHEMATICAL MODELLING COMPONENT
CMA	7	2	$12\frac{1}{2}$	All except MATHEMATICAL MODELLING COMPONENT
TMA	2	0	$12\frac{1}{2}$	MATHEMATICAL MODELLING COMPONENT
Examination	1	–	50	All except MATHEMATICAL MODELLING COMPONENT
Total			100	

This course uses the substitution rule as defined in the Assignments and Examinations section of the *Student Handbook*. The number given in the column headed 'number substitutable' shows the number of assignments for each component which may be replaced by the substitution score.

Note that there is NO substitution for the TMAs associated with the MATHEMATICAL MODELLING COMPONENT.

 SUP 25511 3

6.2

Audio-tapes

To study this course you need an audio cassette-tape player. Instructions for using the tapes are contained in the relevant correspondence texts. The STUDY GUIDE for each correspondence text (in the introduction to the unit) will tell you whether or not that particular unit includes a tape section.

Calculator

You will need a scientific calculator for some of the units, including *Unit 1*. If you have studied *M101*, *MS284*, *MS283*, *TM282* or *TM281*, the calculator you had there will be suitable. If not, you should obtain one which has the basic mathematical functions such as e^x, $\sin x$, $\arcsin x$ and so on, together with a memory.

Computing

The course includes the use of a computer for practical work on numerical analysis at SUMMER SCHOOL. This work relates to *Units 1*, *2*, *9*, *19* and *21*. No programming skills are required. All the computing work is based on specifically prepared programs, called *packages*, which will enable you to solve a variety of problems quickly and efficiently. Instruction on how to use the computer and the packages will be given at Summer School.

Some of the correspondence texts contain sections dealing with computing and computer packages; these should be ignored. (Previously, the computing work was done at the same time as the appropriate unit was being studied, using computer terminals located in study centres. These terminals have now been withdrawn, and all of the computing is done at Summer School. The sections on computing in the correspondence texts are no longer relevant.)

Any arrangements made to help students who have Summer School exemption with the computing component will be announced in a Stop Press.

Continuous assessment only

This term refers to certain parts of the course upon which no questions will be set in the final examination, even though TMA and/or CMA questions may be set on them. For example, the COMPUTING work is of this type. A list of the material which is currently subject to continuous assessment only will be circulated in one of the Stop Presses.

Correspondence texts

There are correspondence texts (units) of the usual kind for each study week, except that there are no units 10, 13, 16 and 23. The first unit of the course, *Unit P*, is a PREPARATORY UNIT which is designed to enable you to revise and practise the mathematical skills from your prerequisite course that you will require in *MST204*. In addition to the numbered units there are two texts associated with the MATHEMATICAL MODELLING COMPONENT: *Unit M: Mathematical modelling* and the *Guide to mathematical modelling*, which you will study at appropriate points in the course, as shown in the Study Calendar.

The texts are divided into sections, each of which we hope you will be able to do in about two hours. Most units have five sections, of which the last is often intended for consolidation and practice of what you have learned in the unit so far, rather than for introducing new material.

The correspondence texts contain various types of questions for you to answer as part of your study. You should try all of them as you come to them (except END-OF-SECTION and END-OF-UNIT EXERCISES and any described as optional) and compare your solutions with the ones given. The questions are of two types, as follows.

Exercises are designed to give you practice in what the preceding text teaches. Solutions are given at the end of the correspondence text.

Problems are more taxing exercises which may take half an hour or so to do, and may draw upon what you have learned in several different parts of the unit, or of the course. Solutions are given at the end of the correspondence text.

The correspondence texts also contain worked *Examples*, which are similar in content to exercises, but which you are not expected to try to do yourself. The solution to an example is given in the text immediately below it.

End-of-section and end-of-unit exercises and problems

These questions, coming after the relevant summaries rather than before, are provided in case you need extra practice to master the work of the unit. You can also use them for examination revision. You are not expected to work through them as you work through the unit for the first time.

Examination

See ASSESSMENT, HANDBOOK.

Examples

See CORRESPONDENCE TEXTS.

Exercises

See CORRESPONDENCE TEXTS.

Graph paper

You will need to provide yourself with suitable graph paper for some of the units.

Handbook

This is provided to give you a convenient source of basic definitions and formulas for use throughout the year and in the examination. You will be allowed to take the *Handbook* into the examination, with any annotations you wish to add, but you must not insert any extra pages. Since you will not be allowed to take any other correspondence texts into the examination, we recommend that from the first you make the *Handbook* your principal source of reference information. There are entries in the *Handbook* for all the units of the course except *Unit P* and *Unit M*.

Keeping to schedule

It is important to keep to schedule for two reasons. One is that the television programmes are closely linked to the work in the correspondence texts, and you will not get the most out of them unless you have done the necessary preparatory work. The other is that you will lose marks if you miss any assignments. For most assignments, the cut-off date is very soon after the end of the study week for the last of the relevant units. We recommend that you finish the assignment questions for each unit as soon as you finish the unit, otherwise you will have a lot of work to do in a few days before the cut-off date.

If you have not done all the work in time for an assignment cut-off date, you should still submit as much of the assignment as you can do, and *start the next unit on time.* As a matter of survival, it is more important to start each unit on time than to do every assignment question.

If it becomes apparent during your study of a unit that you will not have time to do all the work in it, you will have to make some decisions about which parts to leave out. *Such omissions will, in general, cost you marks*, but it is better to lose marks than to get hopelessly behind and drop out. The scheme below (see PARTS OF UNITS WHICH CAN BE OMITTED) indicates the parts of some units which you can omit with least damage. If you have to use it, you should also contact your tutor for advice.

Before using the scheme, you should also consult the Stop Presses for the course to find out which parts of it have been designated for CONTINUOUS ASSESSMENT ONLY. Omitting these parts of the course will not cost you marks on the final examination, though it may cost you TMA and/or CMA marks.

Mathematical modelling component

This is an important component of the course, whose aim is to teach you the skills of mathematical modelling and to give you the opportunity to work on a mathematical model yourself. Mathematical modelling is the art of using mathematical methods to solve real-world problems. The skills required are much wider than those which one normally associates with mathematics; they include analysing the problem, formulating it in mathematical terms, collecting necessary data, interpreting mathematical results, and communicating one's findings.

The mathematical modelling component is distributed through the course. The first instalment takes place about a third of the way through, and is devoted to introducing and developing modelling skills. There is a CORRESPONDENCE TEXT devoted to this aim, *Unit M: Mathematical modelling*; your work on this unit is assessed in TMA 04. The second instalment is encountered mainly at SUMMER SCHOOL, where you will work on modelling a specified problem, in groups with fellow students and with guidance from tutors. There will be preparatory work for you to do before you go to Summer School, the main part of which will be reading the *Guide to mathematical modelling*, a short text designed to help you with your modelling work. When you get back from Summer School you will extend your modelling work and write up your results; this is the third and final instalment of the component. Your report on your modelling activities will be submitted as the second modelling TMA, which is TMA 07.

The precise timing of these activities may vary from year to year, so you should consult the Study Calendar for the relevant information. Note that the TMAs on the modelling component are not substitutable. If you are granted exemption from Summer School, you should consult the Stop Presses for information on the arrangements that have been made to help you with the modelling component.

Parts of units which can be omitted with least damage

Before using this scheme, you should read the instructions in the last two paragraphs of the entry KEEPING TO SCHEDULE.

Unit 1: Recurrence relations Section 2 on second-order recurrence relations. (This will entail omitting Subsection 5.3 of *Unit 5*, which completes the discussion of this topic.)

Unit 2: Differential equations I The exercises in Subsection 3.4 on partial fractions; Section 5.

Unit 3: Animal populations Section 3 on fishing; Section 4 on modelling of exploitation.

Unit 4: Newtonian mechanics in one dimension This unit is fundamental to the later units on mechanics, so try to do it all if you possibly can. But if you are desperate, you would probably do best to reduce your study time on the exercises in Section 4 by reading the solutions instead of trying the exercises yourself.

Unit 5: Complex numbers Subsection 2.2 on number pairs; Section 5 on complex roots of equations and their applications.

Unit 6: Differential equations II Section 5 on a numerical method.

Unit 7: Oscillations and energy Subsection 4.1 on small oscillations, and the associated part of the television programme.

Unit 8: Damped and forced vibrations Try not to omit any of this unit, although if you miss it out, it will not seriously affect your study of the rest of the course. If you are short of time omit Section 4 (TV section).

Unit 9: Simultaneous linear algebraic equations Try not to omit any of this unit.

Unit 12: Heat transfer Subsection 4.2 on cooling from finned pipes; Subsection 4.3 on the combined heat transfer coefficient. Omitting this unit will not affect your study of later units, although it may be useful for your modelling work.

Unit 14: Vector algebra Subsection 3.8 (TV and Tape section).

Unit 15: Newtonian mechanics in three dimensions Section 5 on pendulums.

Unit 17: The dynamics of many-particle systems Section 5 on the motion of a rocket.

Unit 18: Polynomial approximations The sections on the Newton–Raphson method and on interpolation.

Unit 19: Numerical methods for differential equations Subsections 4.2 and 4.3 on Simpson's method, although any part of this unit can be omitted without damage to the rest of the course.

Unit 20: Matrix algebra and determinants Section 3 on change of axes.

Unit 21: Eigenvalues and eigenvectors Sections 2 to 4 can be omitted without damage to the rest of the course.

Unit 22: Simultaneous differential equations A short route through this unit, providing enough knowledge for the rest of the course, is to study Section 1, Subsection 2.1 and Section 3.

Unit 24: Normal modes If you omit Sections 3, 4 and 5 it will not affect the rest of the course.

Unit 25: Functions of more than one variable Section 4 and Subsection 2.3.

Unit 26: Vector calculus Try not to omit any of this unit, but omit Section 4 if you are desperate.

Unit 27: Multiple integrals Sections 3 and 4; but if you are planning to take a third-level applied mathematics course, you are likely to need the material in these sections.

Unit 28: Moments and circular motion Try not to omit any part of this unit as it is a prerequisite for *Units 29* and *30*.

Unit 29: Angular momentum and rigid bodies Sections 3 and 4, although only Section 1 is a necessary prerequisite for the study of *Unit 30*.

Unit 30: Planetary orbits This unit is not a prerequisite for later units. However, do as much as you have time for.

Unit 31: Fourier analysis If you intend to study *Unit 32*, you should study this unit according to the advice given in its STUDY GUIDE.

Unit 32: Partial differential equations Do as much as you have time for. The essential mathematical technique is in Section 3. This material will be useful for third-level applied mathematics courses.

Preparatory unit

This unit (*Unit P*) is designed to ensure that you know all the relevant mathematical techniques from your prerequisite course before beginning work on the new material in *Unit 1*. This unit is assessed, in the first TMA; special arrangements for the speedy marking of the part of that TMA devoted to *Unit P* are described in the first Assignment Booklet.

The preparatory unit has an associated audio-tape, but no television programme. You may therefore begin studying it as soon as you like, and you are strongly urged to devote as much time to it as you can.

Prerequisites

The prerequisite for this course is any one of *M101*, *MS284*, *MS283*, *TM282* and *TM281*.

If you have studied *M101*, you will probably find *Unit 5: Complex numbers* fairly easy because it is very similar to *Unit 1* of *M101* Block VI, and you should find *Unit 20: Matrix algebra* easy because of the matrix algebra in Block IV of *M101*.

If you have studied *MS284*, you should find *Units 5* and *14* of *MST204* relatively straightforward.

If you have studied *MS283*, you will probably find *Units 14* and *26* on vector algebra and vector calculus fairly easy because of the similar units in Block IV of *MS283*.

If you have studied *TM282* or *TM281*, you will find that *Unit 3* on population models, *Unit 7* on oscillations and *Unit 12* on heat transfer contain topics you have already studied.

Problems

See CORRESPONDENCE TEXTS.

Questions

See CORRESPONDENCE TEXTS.

Radio

This course has no radio broadcasts.

Starting the course

Do not wait for the first television programme before starting the course. You are strongly advised to spend as much time on *Unit P*, the PREPARATORY UNIT, as you can. When you have finished with *Unit P* you can, if you want to get a good start, do most of *Unit 1* and Sections 3 and 4 in *Unit 2* before watching any of the television programmes.

Study guide

You will find a paragraph or so with this title in the introduction to each unit. You should read it as soon as you are ready to plan your work on the unit. It will tell you, for example, how much you can vary the order of the sections, and which of them it is essential to study before watching the TELEVISION programme.

Summer School

The course includes a residential Summer School. Besides giving you an opportunity for concentrated study and revision of the mathematical side of the course, it includes some experimental work in mechanics, COMPUTING work, and work on the MATHEMATICAL MODELLING COMPONENT. Arrangements for covering these latter aspects of the Summer School programme for students with exemption will be announced in a Stop Press.

Television

Almost every unit has a television programme, and contains a 'television section' (or subsection) consisting of some preparatory work, a summary of the programme, and some work which follows on directly from the programme. If possible, try to do all this work in a single session.

Plan your work, with the help of the STUDY GUIDE, so as to derive the most benefit from the television section of each unit by making sure that you study all the necessary sections or subsections within that unit before the time of the broadcast. Even when the television section is printed in the latter part of the text, it is often possible to study this section without having studied everything that precedes it.

If you have a video recorder, you may find it helpful to record the programme so that you can view the programme again later in the study week.

You should watch all the programmes if at all possible; but if you do miss one, the relevant part of the correspondence text will still make sense, and you should work through it.

Tutorials

You will hear from the Regional Centre about face-to-face tutorial arrangements.

Unit titles

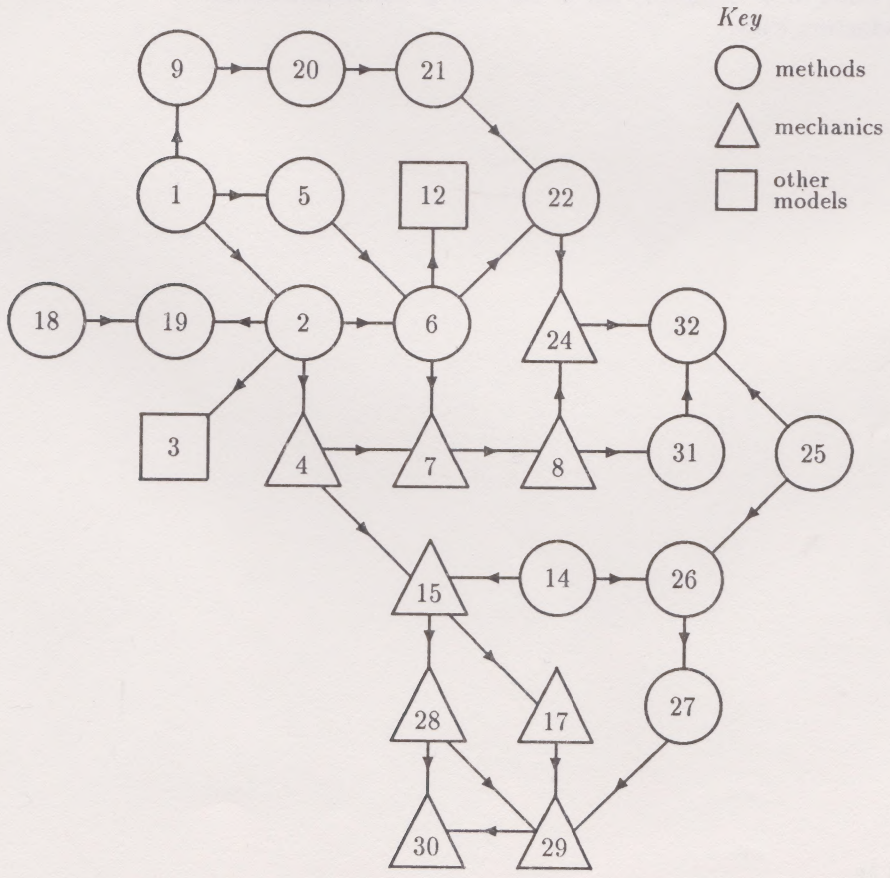
P. Preparatory unit
1. Recurrence relations
2. Differential equations I
3. Animal populations: their growth and exploitation
4. Newtonian mechanics in one dimension
5. Complex numbers
6. Differential equations II
7. Oscillations and energy
8. Damped and forced vibrations
9. Simultaneous linear algebraic equations
12. Heat transfer
14. Vector algebra
15. Newtonian mechanics in three dimensions
17. The dynamics of many-particle systems
18. Polynomial approximations
19. Numerical methods for differential equations
20. Matrix algebra and determinants
21. Eigenvalues and eigenvectors
22. Simultaneous differential equations
24. Normal modes
25. Functions of more than one variable
26. Vector calculus
27. Multiple integrals
28. Moments and circular motion
29. Angular momentum and rigid bodies
30. Planetary orbits
31. Fourier analysis
32. Partial differential equations

There are no Units 10, 11, 13, 16 and 23. Five study weeks are devoted to work on mathematical modelling; *Unit M: Mathematical modelling* and the *Guide to mathematical modelling* are the texts for those weeks. See the Study Calendar and current Stop Presses for information about the mathematical modelling study weeks.

Interrelationships of the units of MST204

This course is highly integrated and the contents of the units are strongly interrelated. It is generally not possible to omit units without it affecting your work on other parts of the course. This diagram shows the *main* relationships between the units. For example, if you omit *Unit 6* then it will have serious implications for your study of *Unit 22*, say.

Unit P has not been included in this diagram, since it is linked to practically every unit of the course!



The course was produced by the following team.

Andrew Adamyck	(BBC)	Shirley Hitchcock	(Mathematics)
Frances Anderson	(Course Coordinator)	Fred Holroyd	(Mathematics)
Terry Berreen	(Mathematics/Technology)	John Jaworski	(BBC)
John Berry	(Mathematics)	Mark Kesby	(Graphic Design)
John Bolton	(Science)	Daniel Lunn	(Mathematics)
David Brannan	(Mathematics)	Tim O'Shea	(IET)
Nick Brenton	(BBC)	Oliver Penrose	(Course Team Chairman:
David Broadhurst	(Science)		Mathematics)
Mick Bromilow	(Mathematics)	Jennifer Phillips	(Mathematics)
Gordon Burt	(IET)	Graham Read	(Mathematics)
Alison Cadle	(Publishing)	John Richmond	(BBC)
Paul Clark	(Science)	Chris Rowley	(Mathematics)
Bob Coates	(Mathematics)	Mike Simpson	(Mathematics)
Priscilla Coley	(Course Team Secretary)	Ted Smith	(BBC)
Jo Cotter	(Graphic Design)	Tom Smith	(Science)
Dick Crabbe	(Publishing)	Allan Solomon	(Mathematics)
Mike Crampin	(Mathematics)	Pip Surgey	(BBC)
Margaret Crowe	(Course Manager)	Peter Taylor	(Consultant)
Judith Daniels	(Mathematics)	Frances Thomas	(Course Team Secretary)
Roger Duke	(Mathematics)	Glanffrwd Thomas	(BBC)
Tony Edwards	(Consultant)	Howard Thomas	(Mathematics)
Pat Edwin	(Mathematics)	Peter Thomas	(Mathematics)
Judy Ekins	(Mathematics)	Mike Thorpe	(Mathematics)
Matthew Esplen	(Mathematics)	John Trapp	(Mathematics)
Dick Fendrich	(Technology)	Bob Tunnicliffe	(Mathematics)
Norman Gowar	(Mathematics)	Mirabelle Walker	(Consultant)
Joel Greenberg	(ACS)	Ros Wood	(Graphic Design)
Garry Hammond	(Publishing)	Martin Wright	(BBC)

Particular thanks are due to the academic editors: Judith Daniels, Roger Duke, Matthew Esplen, Fred Holroyd and Mike Simpson; and to the staff of the Mathematics Faculty Course Materials Production Unit.

Printed in the United Kingdom by
The Open University